***University Physics Volume I***

**Unit 2: Waves and Acoustics**

**Chapter 15: Oscillations**

**Conceptual Questions**

1. What conditions must be met to produce SHM?

Solution

The restoring force must be proportional to the displacement and act opposite to the direction of motion with no drag forces or friction. The frequency of oscillation does not depend on the amplitude.

1. (a) If frequency is not constant for some oscillation, can the oscillation be SHM? (b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

Solution

(a) If the frequency is not constant, some change in force or additional force must have changed the frequency, which suggests that the system is not oscillating in simple harmonic motion.

(b) No, the frequency of a system in simple harmonic motion depends on the force constant and the mass.

1. Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

Solution

Examples: Mass attached to a spring on a frictionless table, a mass hanging from a string, a simple pendulum with a small amplitude of motion. All of these examples have frequencies of oscillation that are independent of amplitude.

1. Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a more pliable material.

Solution

A stiff material would have a stronger force constant, and a stronger restoring force for a given displacement. Since the frequency of oscillations is equal to  the frequency would be higher.

1. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

Solution

Since the frequency is proportional to the square root of the force constant and inversely proportional to the square root of the mass, it is likely that the truck is heavily loaded, since the force constant would be the same whether the truck is empty or heavily loaded.

1. Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

Solution

A stiffer spring will produce a smaller amplitude of oscillation. Not replacing the springs could result in the car hitting the ground.

1. Describe a system in which elastic potential energy is stored.

Solution

In a car, elastic potential energy is stored when the shock is extended or compressed. In some running shoes elastic potential energy is stored in the compression of the material of the soles of the running shoes. In pole vaulting, elastic potential energy is stored in the bending of the pole

1. Explain in terms of energy how dissipative forces such as friction reduce the amplitude of a harmonic oscillator. Also explain how a driving mechanism can compensate. (A pendulum clock is such a system.)

Solution

Consider a mass on a spring sliding on a frictionless table. When the spring is extended or compressed, elastic potential energy is stored in the spring. Since there are no dissipative forces, the energy oscillates between potential energy and kinetic energy. The restoring force always acts in the opposite direction of the displacement. If there is friction between the mass and the table, the friction always acts in the opposite direction of the velocity, dissipating the energy of the system and decreasing the amplitude of the motion. Using a driving mechanism can add energy to the system, doing work on the system, as it is being dissipated by the friction.

1. The temperature of the atmosphere oscillates from a maximum near noontime and a minimum near sunrise. Would you consider the atmosphere to be in stable or unstable equilibrium?

Solution

The overall system is stable. There may be times when the stability is interrupted by a storm, but the driving force provided by the sun bring the atmosphere back into a stable pattern.

1. Can this analogy of SHM to circular motion be carried out with an object oscillating on a spring vertically hung from the ceiling? Why or why not? If given the choice, would you prefer to use a sine function or a cosine function to model the motion?

Solution

Yes, the analogy can be used for a mass oscillating vertically, but using the *y* component instead of the *x* component. The choice of cosine or sine should be regulated by the initial conditions. If the initial velocity is a positive maximum, and position is zero at time use a sine function. If the initial velocity is a zero, and position is a positive maximum at time use a cosine function. Since an initial phase shift can be included, either sine or cosine can be used in any situation. The analysis will still yield  .

1. If the maximum speed of the mass attached to a spring, oscillating on a frictionless table, was increased, what characteristics of the rotating disk would need to be changed?

Solution

The maximum speed is equal to  and the angular frequency is independent of the amplitude, so the amplitude would be affected. The radius of the circle represents the amplitude of the circle, so make the amplitude larger.

1. Pendulum clocks are made to run at the correct rate by adjusting the pendulum’s length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

Solution

Yes, it will have to be changed. The frequency needs to remain constant and the angular frequency of a simple pendulum is  If the acceleration due to gravity is slightly greater, the length will need to be increased to keep the angular frequency, and therefore the frequency, constant.

1. A pendulum clock works by measuring the period of a pendulum. In the springtime the clock runs with perfect time, but in the summer and winter the length of the pendulum changes. When most materials are heated, they expand. Does the clock run too fast or too slow in the summer? What about the winter?

Solution

The period of the pendulum is  In summer, the length increases, and the period increases. If the period should be one second, but period is longer than one second in the summer, it will oscillate fewer than 60 times a minute and clock will run slow. In the winter it will run fast.

1. With the use of a phase shift, the position of an object may be modeled as a cosine or sine function. If given the option, which function would you choose? Assuming that the phase shift is zero, what are the initial conditions of function; that is, the initial position, velocity, and acceleration, when using a sine function? How about when a cosine function is used?

Solution

Whatever function is most convenient is the one to use. If the initial position is zero, initial velocity is a positive maximum, and the initial acceleration is zero, use a sine function. If initial position is a positive maximum, the initial velocity is zero, and the acceleration is a negative maximum, use a cosine function. If the initial position is zero, initial velocity is a negative maximum, and the initial acceleration is zero, use a negative sine function. If initial position is a negative maximum, the initial velocity is zero, and the acceleration is a positive maximum, use a negative cosine function. Really, because of the initial phase shift, it does not matter what you choose as long as you choose the proper phase shift.

1. Give an example of a damped harmonic oscillator. (They are more common than undamped or simple harmonic oscillators.)

Solution

A car shock absorber.

1. How would a car bounce after a bump under each of these conditions?
2. overdamping
3. underdamping
4. critical damping

Solution

If overdamped, the car absorber will not oscillate and will not return to the equilibrium position when disturbed. If critically damped, the shock will not oscillate, but will return to equilibrium position when disturbed. If underdamped, the car will oscillate, dissipating the energy and eventually come to rest. If there is too little damping, the car may bounce for an uncomfortable amount of time before coming to rest.

1. Most harmonic oscillators are damped and, if undriven, eventually come to a stop. Why?

Solution

The second law of thermodynamics states that perpetual motion machines are impossible. Eventually the ordered motion of the system decreases and returns to equilibrium.

1. Why are soldiers in general ordered to “route step” (walk out of step) across a bridge?

Solution

If the soldiers are all walking in step, the driving force may cause the bridge to resonate.

1. Do you think there is any harmonic motion in the physical world that is not damped harmonic motion? Try to make a list of five examples of undamped harmonic motion and damped harmonic motion. Which list was easier to make?

Solution

All harmonic motion is damped harmonic motion, but the damping may be negligible. This is due to friction and drag forces. It is easy to come up with five examples of damped motion: (1) A mass oscillating on a hanging on a spring (it eventually comes to rest). (2) Shock absorbers in a car (thankfully they also come to rest). (3) A pendulum is a grandfather clock (weights are added to add energy to the oscillations). (4) A child on a swing (eventually comes to rest unless energy is added by pushing the child). (5) A marble rolling in a bowl (eventually comes to rest). As for the undamped motion, even a mass on a spring in a vacuum will eventually come to rest due to internal forces in the spring. Damping may be negligible, but cannot be eliminated.

1. Some engineers use sound to diagnose performance problems with car engines. Occasionally, a part of the engine is designed that resonates at the frequency of the engine. The unwanted oscillations can cause noise that irritates the driver or could lead to the part failing prematurely. In one case, a part was located that had a length *L* made of a material with a mass *M*. What can be done to correct this problem?

Solution

Possibly the length can be adjusted slightly or the density of the material can be adjusted by changing the chemical compound of the part; for example, if composed of metal, a slightly different alloy can be used.

**Problems**

1. Prove that using  will produce the same results for the period for the oscillations of a mass and a spring. Why do you think the cosine function was chosen?

Solution

Proof

1. What is the period of 60.0 Hz of electrical power?

Solution

16.7 ms

1. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

Solution

0.400 s/beat

1. Find the frequency of a tuning fork that takes  to complete one oscillation.

Solution

400 Hz

1. A stroboscope is set to flash every . What is the frequency of the flashes?

Solution

12,500 Hz

1. A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

Solution

1500 Hz

1. Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

Solution

a. 340 km/hr; b. rev/min

1. A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass?

Solution

2.37 N/m

1. A mass  is attached to a spring and hung vertically. The mass is raised a short distance in the vertical direction and released. The mass oscillates with a frequency . If the mass is replaced with a mass nine times as large, and the experiment was repeated, what would be the frequency of the oscillations in terms of ?

Solution



1. A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

Solution

0.389 kg

1. How much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?

Solution

0.009 kg; 2%

1. Fish are hung on a spring scale to determine their mass. (a) What is the force constant of the spring in such a scale if it the spring stretches 8.00 cm for a 10.0 kg load? (b) What is the mass of a fish that stretches the spring 5.50 cm? (c) How far apart are the half-kilogram marks on the scale?

Solution

a. ; b. 6.88 kg; c. 4.00 mm

1. It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke’s law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring’s effective force constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85-kg team?

Solution

a. ; b. 77 kg, yes, he is eligible to play

1. One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger’s spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?

Solution

a. 889 N/m; b. 133 N

1. When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m. (a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

Solution

a. ; b. yes, when the man is at his lowest point in his hopping the spring will be compressed the most

1. A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

Solution

a. 29.4 N/m; b. 10.0 cm

1. The length of nylon rope from which a mountain climber is suspended has an effective force constant of . (a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg? (b) How much would this rope stretch to break the climber’s fall if he free-falls 2.00 m before the rope runs out of slack? *Hint:* Use conservation of energy. (c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

Solution

a. 1.99 Hz; b. 56.9 cm; c. 77.6 cm

1. The motion of a mass on a spring hung vertically, where the mass oscillates up and down, can also be modeled using the rotating disk. Instead of the lights being placed horizontally along the top and pointing down, place the lights vertically and have the lights shine on the side of the rotating disk. A shadow will be produced on a nearby wall, and will move up and down. Write the equations of motion for the shadow taking the position at  to be  with the mass moving in the positive *y*-direction.

Solution

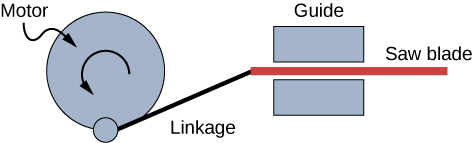


1. (a) A novelty clock has a 0.0100-kg-mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

Solution

a. 0.335 m/s; b. 

1. Reciprocating motion uses the rotation of a motor to produce linear motion up and down or back and forth. This is how a reciprocating saw operates, as shown below.



If the motor rotates at 60 Hz and has a radius of 3.0 cm, estimate the maximum speed of the saw blade as it moves left and right. This design is known as a scotch yoke.

Solution



1. A student stands on the edge of a merry-go-round which rotates five times a minute and has a radius of two meters one evening as the sun is setting. The student produces a shadow on the nearby building. (a) Write an equation for the position of the shadow. (b) Write an equation for the velocity of the shadow.

Solution

a. ; b. 

1. What is the length of a pendulum that has a period of 0.500 s?

Solution

6.21 cm

1. Some people think a pendulum with a period of 1.00 s can be driven with “mental energy” or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

Solution

24.8 cm

1. What is the period of a 1.00-m-long pendulum?

Solution

2.01 s

1. How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

Solution

4.01 s

1. The pendulum on a cuckoo clock is 5.00-cm long. What is its frequency?

Solution

2.23 Hz

1. Two parakeets sit on a swing with their combined CMs 10.0 cm below the pivot. At what frequency do they swing?

Solution

1.58 s

1. (a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is  is moved to a location where the acceleration due to gravity is . What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.

Solution

a. 2.99541 s; b. Since the period is related to the square root of the acceleration of gravity, when the acceleration changes by 1%, the period changes by  so it is necessary to have four digits to see the changes.

1. A pendulum with a period of 2.00000 s in one location () is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

Solution



1. (a) What is the effect on the period of a pendulum if you double its length? (b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

Solution

a. ; b. 97.5T

1. The amplitude of a lightly damped oscillator decreases by  during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Solution

6%

1. How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

Solution

3.84 J

1. If a car has a suspension system with a force constant of , how much energy must the car’s shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

Solution

141 J

1. (a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

Solution

a. 0.123 m; b. –0.600 J; c. 0.300 J; The rest of the energy may go into heat caused by friction and other damping forces.

1. Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction . (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is , what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

Solution

a. ; b. 

**Additional Problems**

1. Suppose you attach an object with mass *m* to a vertical spring originally at rest, and let it bounce up and down. You release the object from rest at the spring’s original rest length, the length of the spring in equilibrium, without the mass attached. The amplitude of the motion is the distance between the equilibrium position of the spring without the mass attached and the equilibrium position of the spring with the mass attached. (a) Show that the spring exerts an upward force of 2.00*mg* on the object at its lowest point. (b) If the spring has a force constant of 10.0 N/m, is hung horizontally, and the position of the free end of the spring is marked as , where is the new equilibrium position if a 0.25-kg-mass object is hung from the spring? (c) If the spring has a force constant of 10.0 N/m and a 0.25-kg-mass object is set in motion as described, find the amplitude of the oscillations. (d) Find the maximum velocity.

Solution

a. ; b. –24.5 cm; c. 0.245 m; d. 1.55 m/s

1. A diver on a diving board is undergoing SHM. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

Solution

94.7 kg

1. Suppose a diving board with no one on it bounces up and down in a SHM with a frequency of 4.00 Hz. The board has an effective mass of 10.0 kg. What is the frequency of the SHM of a 75.0-kg diver on the board?

Solution

1.37 Hz

1. The device pictured in the following figure entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring. (a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its force constant? (b) What is the time for one complete bounce of this child? (c) What is the child’s maximum velocity if the amplitude of her bounce is 0.200 m?



(credit: Lisa Doehnert)

Solution

a. 314 N/m; b. 1.00 s; c. 1.25 m/s

1. A mass is placed on a frictionless, horizontal table. A spring  which can be stretched or compressed, is placed on the table. A 5.00-kg mass is attached to one end of the spring, the other end is anchored to the wall. The equilibrium position is marked at zero. A student moves the mass out to  and releases it from rest. The mass oscillates in SHM. (a) Determine the equations of motion. (b) Find the position, velocity, and acceleration of the mass at time 

Solution

a. ; b. 

1. Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is .

Solution

ratio of 2.45

1. At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is , if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock’s hour hand to make one revolution on the Moon.

Solution

The clock will run slow by a factor of 2.45.

1. If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

Solution

The length must increase by 0.0116%.

1. A 2.00-kg object hangs, at rest, on a 1.00-m-long string attached to the ceiling. A 100-g mass is fired with a speed of 20 m/s at the 2.00-kg mass, and the 100.00-g mass collides perfectly elastically with the 2.00-kg mass. Write an equation for the motion of the hanging mass after the collision. Assume air resistance is negligible.

Solution



1. A 2.00-kg object hangs, at rest, on a 1.00-m-long string attached to the ceiling. A 100-g object is fired with a speed of 20 m/s at the 2.00-kg object, and the two objects collide and stick together in a totally inelastic collision. Write an equation for the motion of the system after the collision. Assume air resistance is negligible.

Solution



1. Assume that a pendulum used to drive a grandfather clock has a length  and a mass *M* at temperature  It can be modeled as a physical pendulum as a rod oscillating around one end. By what percentage will the period change if the temperature increases by  Assume the length of the rod changes linearly with temperature, where  and the rod is made of brass 

Solution

0.009%

1. A 2.00-kg block lies at rest on a frictionless table. A spring, with a spring constant of 100 N/m is attached to the wall and to the block. A second block of 0.50 kg is placed on top of the first block. The 2.00-kg block is gently pulled to a position  and released from rest. There is a coefficient of friction of 0.45 between the two blocks. (a) What is the period of the oscillations? (b) What is the largest amplitude of motion that will allow the blocks to oscillate without the 0.50-kg block sliding off?

Solution

a. 0.99 s; b. 0.11 m

**Challenge Problems**

1. A suspension bridge oscillates with an effective force constant of . (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge’s natural frequency and impart  of energy each second, how long does it take for the bridge’s oscillations to go from 0.100 m to 0.500 m amplitude.

Solution

a. ; b. 

1. Near the top of the Citigroup Center building in New York City, there is an object with mass of  on springs that have adjustable force constants. Its function is to dampen wind-driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement from equilibrium?

Solution

a. ; b. 

1. Parcels of air (small volumes of air) in a stable atmosphere (where the temperature increases with height) can oscillate up and down, due to the restoring force provided by the buoyancy of the air parcel. The frequency of the oscillations are a measure of the stability of the atmosphere. Assuming that the acceleration of an air parcel can be modeled as , prove that  is a solution, where *N* is known as the Brunt-Väisälä frequency. Note that in a stable atmosphere, the density decreases with height and parcel oscillates up and down.

Solution

It is true if, 

1. Consider the van der Waals potential  , used to model the potential energy function of two molecules, where the minimum potential is at . Find the force as a function of *r*. Consider a small displacement  and use the binomial theorem:

,

to show that the force does approximate a Hooke’s law force.

Solution



1. Suppose the length of a clock’s pendulum is changed by 1.000%, exactly at noon one day. What time will the clock read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.

Solution

11:52:51 A.M.; 12:07:15 P.M.

1. (a) The springs of a pickup truck act like a single spring with a force constant of . By how much will the truck be depressed by its maximum load of 1000 kg? (b) If the pickup truck has four identical springs, what is the force constant of each?

Solution

a. 7.54 cm; b. 

This file is copyright 2016, Rice University. All Rights Reserved.